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ABSTRACT

We formulate boundary conditions for an open membrane that ends on the fivebrane of \mathcal{M} -theory. We show that the dynamics of the eleven-dimensional fivebrane can be obtained from the quantization of a “small membrane” that is confined to a single fivebrane and which moves with the speed of light. This shows that the eleven-dimensional fivebrane has an interpretation as a D -brane of an open supermembrane as has recently been proposed by Strominger and Townsend. We briefly discuss the boundary dynamics of an infinitely extended planar membrane that is stretched between two parallel fivebranes.

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1. Introduction

\mathcal{M} -theory is the “magic” quantum theory in eleven dimensions from which properties of string theory, such as duality symmetries, can be understood in a unified framework. The low energy limit of this theory, eleven dimensional supergravity, contains membranes and fivebranes. Properties of \mathcal{M} -theory can be analyzed, for example, by considering compactifications of these extended objects to lower dimensions[‡].

All the ten-dimensional type IIA p -branes can be related to either the membrane or the fivebrane [9] of eleven dimensional supergravity. Polchinski [10] has shown that ten-dimensional Ramond-Ramond (RR) p -branes can be described as D -branes[§], that are hyperplanes on which an open string is allowed to end [12]. The appearance of Dirichlet boundary conditions can be naturally understood as a consequence of T -duality [12]. String duality therefore teaches us that we have to incorporate Dirichlet boundary conditions in open string theory in order to have a consistent theory. Moreover, as explained in [12], a D -brane is a dynamical object. Massless open string excitations that propagate on the D -brane have an interpretation as collective coordinates for the transverse fluctuations of the D -brane. The recent work of Douglas [13], Strominger [14], Townsend [15] and Vafa [16] showed that p -branes can have boundaries on p -branes. More concretely, Strominger [14] and Townsend [15] suggested that the fivebrane may have an interpretation as a D -brane of an open membrane in eleven dimensions.

One problem that we would like to address in this paper is to show that the dynamics of the fivebrane can be obtained from the quantization of a “small membrane” after imposing appropriate boundary conditions on the fields. In section 2 we formulate boundary conditions for an open membrane whose boundaries lie on the fivebrane of \mathcal{M} -theory. For this purpose we will use the light-cone gauged

[‡] See [1,2,3,4,5,6,7,8] and references therein.

[§] A recent review article is [11].

fixed action of the supermembrane. We will consider two particular classical configurations in the next two sections. In section 3 we show that the massless fields on the worldvolume of the fivebrane can be obtained from the quantization of the fermionic zero-modes of a pointlike collapsed membrane that is confined to the fivebrane. In section 4 we discuss the boundary dynamics of an infinitely extended planar membrane that is stretched between two parallel fivebranes. In appendix A we use the Siegel gauge to show that the fermionic zero-modes exhibit the full $SO(5,1)$ chirality of the worldvolume fields of the fivebrane. Our notation and conventions are explained in appendix B.

2. Boundary Conditions for the Open Membrane

The action for the eleven-dimensional supermembrane in flat superspace is given by [17,18]:

$$\begin{aligned} \mathcal{S} = -\frac{1}{2} \int d^3\sigma \sqrt{-h} \Big[& h^{\alpha\beta} \Pi_\alpha^M \Pi_{\beta M} - 1 + i\epsilon^{\alpha\beta\gamma} \bar{\Theta} \Gamma_{MN} \partial_\alpha \Theta \left(\Pi_\beta^M \Pi_\gamma^N \right. \\ & \left. + i\Pi_\beta^M \bar{\Theta} \Gamma^N \partial_\gamma \Theta - \frac{1}{3} \bar{\Theta} \Gamma^M \partial_\beta \Theta \bar{\Theta} \Gamma^N \partial_\gamma \Theta \right) \Big], \end{aligned} \quad (2.1)$$

In the above expression Θ is a 32-component Majorana spinor, $X^M(\sigma)$, with $M = 0, \dots, 10$, describes the bosonic membrane configuration and $h_{\alpha\beta}$, with $\alpha, \beta = 0, 1, 2$, is an auxiliary worldbrane metric with Minkowski signature. Furthermore

$$\Pi_\alpha^M = \partial_\alpha X^M - i\bar{\Theta} \Gamma^M \partial_\alpha \Theta. \quad (2.2)$$

Henceforth we neglect higher order terms in fermionic variables.

This action is invariant under worldvolume general coordinate transformations and under Poincaré transformations in the eleven-dimensional Minkowski space-time. Up to surface terms (whose cancellation depends on the boundary conditions

chosen) it is also invariant under global space-time supersymmetries:

$$\begin{aligned}\delta_\epsilon \Theta &= \epsilon, \\ \delta_\epsilon X^M &= i\bar{\epsilon}\Gamma^M \Theta,\end{aligned}\tag{2.3}$$

as well as local κ -symmetries

$$\begin{aligned}\delta_\kappa \Theta &= 2P_+ \kappa(\sigma), \\ \delta_\kappa X^M &= 2i\bar{\Theta}\Gamma^M P_+ \kappa(\sigma).\end{aligned}\tag{2.4}$$

Here ϵ and $\kappa(\sigma)$ are eleven-dimensional Majorana spinors that are worldvolume scalars. P_\pm are projection operators [19] defined by

$$P_\pm = \frac{1}{2} \left(1 \pm \frac{1}{3!} \epsilon^{\alpha\beta\gamma} \partial_\alpha X^M \partial_\beta X^N \partial_\gamma X^P \Gamma_{MNP} \right).\tag{2.5}$$

The equations of motion are the embedding equation for the worldbrane metric [18,20]:

$$h_{\alpha\beta} = \partial_\alpha X^M \partial_\beta X_M,\tag{2.6}$$

and the brane-wave equations

$$\begin{aligned}\partial_\alpha \left(\sqrt{-h} h^{\alpha\beta} \partial_\beta X^M \right) &= 0, \\ P_- h^{\alpha\beta} \partial_\alpha X^M \Gamma_M \partial_\beta \Theta &= 0.\end{aligned}\tag{2.7}$$

Since P_- is a projection operator, the second equation in (2.7) is an equation of motion for only 16 of the 32 components of Θ . This is due to the fact that the action has a κ -symmetry. The local invariances of the action can be used to impose the string inspired light-cone gauge [21,18]:

$$X^+ = p^+ \tau, \quad h_{\alpha\beta} = \begin{pmatrix} -\det g & 0 \\ 0 & g_{ab} \end{pmatrix} \quad \text{and} \quad \Gamma^+ \Theta = 0,\tag{2.8}$$

where $a, b = 1, 2$. Choosing an adequate representation of the gamma matrices the field Θ can be written in the form $\Theta = (0, S)$, where S is a real 16-component

$SO(9)$ spinor. Thus half of the 32 components of Θ have been eliminated. We can further eliminate three bosonic degrees of freedom. The first equation in (2.8) eliminates X^+ as independent coordinate and inserting the gauge conditions into the embedding equation one finds that X^- satisfies [22,18]:

$$\begin{aligned}\dot{X}^- &= \frac{1}{2p_+} \left(\dot{X}^I \dot{X}_I + \det g \right), \\ \partial_a X^- &= \frac{1}{p_+} \dot{X}^I \partial_a X_I,\end{aligned}\tag{2.9}$$

where $I = 1, \dots, 9$. This determines X^- and furthermore from the second equation one obtains the constraint

$$\epsilon^{ab} \partial_a \dot{X}^I \partial_b X_I = 0,\tag{2.10}$$

which can be used to eliminate an additional coordinate. In the following we will take this variable to be X^1 without loss of generality. Therefore we are left with eight bosonic degrees of freedom. In the light-cone gauge the covariant field equations (2.6) and (2.7) take the form

$$\begin{aligned}g_{ab} &= \partial_a X^I \partial_b X_I, \\ \ddot{X}^I &= \partial_a (g g^{ab} \partial_b X^I), \\ \dot{S} &= -\epsilon^{ab} \partial_a X^I \gamma_I \partial_b S.\end{aligned}\tag{2.11}$$

The light-cone gauge fixed action has residual symmetries, that leave the gauge conditions (2.8) and the equations of motion (2.11) invariant. The form of these transformations has been obtained in [18]. They are the α -symmetries:

$$\begin{aligned}\delta S &= -\frac{1}{\sqrt{2}p^+} \left(\dot{X}^I \gamma_I - \frac{1}{2} \epsilon^{ab} \partial_a X^I \partial_b X^J \gamma_{IJ} \right) \alpha, \\ \delta_\alpha X^I &= 2i\bar{\alpha} \gamma^I S + 2i\epsilon^{ab} \partial_a X^I \bar{\alpha} \int_0^\tau d\tau \partial_b S,\end{aligned}\tag{2.12}$$

and the β -symmetries:

$$\begin{aligned}\delta_\beta S &= \beta, \\ \delta_\beta X^I &= 0.\end{aligned}\tag{2.13}$$

Here α and β are 16-component spinors resulting from the decomposition $\epsilon = (i\alpha, \beta)$. From the above symmetries only the first one is a supersymmetry.

Consider now the variation of the action under a variation of the bosonic fields. Demanding the equation of motion for X^I to hold, the variational principle for the action is:

$$\int_{\partial\Sigma} n^a \partial_a X_I \delta X^I = 0,\tag{2.14}$$

where n^a is the outward-pointing unit normal to the boundary. We will take n^a to be spacelike everywhere. This imposes constraints for the values of the fields at the boundary. For the bosonic fields we could in principle impose either Neumann boundary conditions, where we allow δX^I to be arbitrary and the normal derivative vanishes, or Dirichlet boundary conditions where X^I is held fixed at the boundary. However, it was shown in [14] that RR charge conservation does not allow the existence of free open membranes[★]. Charge conservation allows a membrane ending on one fivebrane. When the fivebrane lies in the hyperplane $X^M = 0$ for $M = 1, \dots, 5$, the boundary conditions for the physical bosonic fields are

$$\begin{aligned}X_{\mathcal{D}}^I &= 0, & I &= 2, 3, 4, 5, \\ n^a \partial_a X_{\mathcal{N}}^I &= 0, & I &= 6, 7, 8, 9, & \text{on } \partial\Sigma.\end{aligned}\tag{2.15}$$

Next we would like to determine the boundary conditions for S^\dagger . The vanishing

★ An exception of this is the open membrane considered in [4], where the boundaries lie at the end of space-time.

† Boundary conditions for the fermionic fields of an open spinning membrane were discussed previously in [23].

of $\delta\mathcal{S}$ under the variation of the fermionic field requires

$$\int_{\partial\Sigma} \bar{S}\not{t}\delta S = 0, \quad (2.16)$$

where $\not{t} = t^a \partial_a X_{\mathcal{N}}^I \gamma_I$. Since the Dirac operator is first order, we are only allowed to fix half of the components of the fermionic field at the boundary. For this purpose we introduce boundary projection operators

$$\wp_{\pm} = \frac{1}{2}(1 \pm \gamma_6 \gamma_7 \gamma_8), \quad (2.17)$$

which act on spinor fields at the boundary. These operators satisfy

$$\wp_{\pm}^2 = \wp_{\pm}, \quad \wp_+ \wp_- = 0 \quad \text{and} \quad \wp_+ + \wp_- = 1. \quad (2.18)$$

The expression (2.16) vanishes if we impose the Dirichlet condition

$$\wp_+ S = 0 \quad \text{on } \partial\Sigma. \quad (2.19)$$

This can be easily seen using (2.17) and the “flipping” property of appendix B. From (2.12) we see that the supersymmetry generating parameter satisfies the boundary condition

$$\wp_- \alpha = 0 \quad \text{on } \partial\Sigma. \quad (2.20)$$

This implies that half of the supersymmetries are broken in the presence of the boundary, as it happens for the open superstring. The above equations state that the fermionic fields have a well defined $SO(4)$ chirality, longitudinal to the fivebrane. This choice of fermionic boundary conditions guarantees that on the boundary $\delta_{\alpha} X_{\mathcal{D}}^I = 0$, while there is no restriction on the Neumann coordinates. This means that the residual supersymmetry transformation respects the Dirichlet boundary conditions on the bosonic fields, as it has to be.

To show that the Neumann boundary conditions are compatible with the residual supersymmetries, we first have to find the boundary conditions for the orthogonal component of the fermionic field. They can be obtained using the Dirac equation and the boundary condition (2.19). Here we have to extend the definition of n^a and t^a to a neighborhood of $\partial\Sigma$ by parallel transport along geodesics normal to $\partial\Sigma$, so that $n^a\nabla_a n^b = n^a\nabla_a t^b$. Then, it is easiest to start with the expression

$$\wp_+ \left(\dot{S} + \epsilon^{ab} \partial_a X^I \gamma_I \partial_b S \right) = 0, \quad (2.21)$$

that can be further transformed taking into account that the derivative of (2.19) with respect to the tangent along the boundary is equal to zero. This leads us to

$$n^a \partial_a (\wp_- S) = 0 \quad \text{on } \partial\Sigma. \quad (2.22)$$

This boundary condition implies that $n^a \partial_a (\delta_\alpha X_\mathcal{N}^I) = 0$ and is therefore compatible with the bosonic Neumann boundary condition in (2.15).

Notice that to derive the boundary conditions on the bosonic and fermionic fields we did not need to specify a particular classical configuration. The specific form of the classical configuration, together with the boundary condition (2.19) determines the number of unbroken supersymmetries for a particular brane configuration. In the following we will consider two particular classical membrane configurations: a collapsed membrane or zero-brane that is confined to a single fivebrane and an infinitely extended planar membrane that is stretched between two parallel fivebranes.

3. The Zerobrane Confined to the Fivebrane

Some time ago, Bars, Pope and Sezgin [22] obtained the massless spectrum of eleven-dimensional supergravity from the quantization of a closed membrane that was completely collapsed to a point. Here we would like to show that the massless fields on the fivebrane worldvolume can be obtained from the quantization of a zerobrane which is confined to the fivebrane and that is traveling with the speed of light. This collapsed membrane can be viewed as the groundstate of an open membrane having boundaries on the fivebrane.[★]

In the limit where the membrane is collapsed to a point the bosonic part of the classical field configuration takes the form

$$X^M(\tau, \sigma_1, \sigma_2) = x^M + p^M \tau, \quad (3.1)$$

where x^M and p^M are constants. For this configuration the metric becomes degenerate, but nevertheless the field equations are well defined, so that this solution is perfectly regular. The equations of motion (2.6) and (2.7) in the light-cone gauge, reduce to the equations of motion of a massless superparticle [22,24],

$$p^2 = 0, \quad \dot{p}^M = 0 \quad \text{and} \quad p_M \Gamma^M \dot{\Theta} = 0, \quad (3.2)$$

which is moving with the speed of light. Due to this fact, the spectrum of the supermembrane contains massless particles. The Dirichlet conditions on the bosonic coordinates (2.15) translate to the constraint

$$X^M \equiv 0 \quad \text{for} \quad M = 2, 3, 4, 5, \quad (3.3)$$

which means that the particle is confined to the fivebrane. The residual supersym-

★ We believe that the topology of the extended membrane whose groundstate is described by this zerobrane is a cylinder in $\mathbb{R}^{10} \times S^1$ which has one end lying on the fivebrane and is closed at the other end. However, further analysis is required to confirm this. This would be relevant to understand the massive spectrum of the theory.

metries for the configuration (3.1) take the form:

$$\begin{aligned}\delta_\alpha S &= -\frac{1}{\sqrt{2}p^+}\dot{X}^I\gamma_I\alpha, \\ \delta_\alpha X^I &= 2i\bar{\alpha}\gamma^I S.\end{aligned}\tag{3.4}$$

Using this α -symmetry and the bosonic constraint (3.3) we obtain the conditions

$$(1 + \gamma_{6789})S = (1 - \gamma_{6789})\alpha = 0.\tag{3.5}$$

Thus the fermionic fields have a well defined $SO(4)$ chirality longitudinal to the fivebrane.

The massless spectrum of the theory can be determined from the quantization of the fermionic zero-modes obtained from (3.4). In the light-cone gauge there are 16 real fermionic zero-modes that are subjected to the constraint (3.5). Therefore only eight linearly independent fermionic zero-modes are left. Since the light-cone gauge fixed action contains the term $\bar{S}\dot{S}$, the canonical conjugate to real S is again S , so that the zero-modes satisfy a Clifford algebra

$$\{S_0^\alpha, S_{0\beta}\} = 2\delta_\beta^\alpha.\tag{3.6}$$

These fermionic zero-modes can be rearranged into four creation and four annihilation operators. The corresponding Hilbert space will be 2^4 -dimensional with 2^3 bosons and 2^3 fermions.

We would like to argue that the zero-modes (3.6) subjected to the constraint (3.5) generate a chiral $N = 2$, $d = 6$ spectrum corresponding to the collective coordinates on the fivebrane worldvolume. There are two $N = 2$ theories in $d = 6$ depending on their field content [25,26,27,28,29]. There are three different matter multiplets, Φ , A and B [27]. Each of these matter multiplets contains a doublet of Weyl fermions. In the $(1,1)$ theory these fermions have different chirality while they have the same for the $(2,0)$ theory. Φ contains four scalars, A a single vector and B contains an anti-self-dual tensor field and a single scalar.

The $(0,2)$ theory has a matter multiplet which is the sum of the Φ and B multiplet. The anti-self-dual tensor has three degrees of freedom which together with the five scalars matches the eight bosonic degrees of freedom that we get out of the representation of the Clifford algebra. The constraint (3.5) indicates that the fermionic zero-modes have a well defined chirality longitudinal to the fivebrane.

Therefore we have generated the chiral $(0,2)$ $N = 2$ theory in the light-cone gauge, corresponding to the massless states on the six-dimensional worldvolume of the fivebrane! This shows that the fivebrane has indeed an interpretation as a D -brane of an open membrane in eleven dimensions.

4. The Membrane Stretched Between Parallel Fivebranes

In addition to a membrane with boundaries on a single fivebrane, we could consider a membrane that is stretched between two (or more) parallel fivebranes. An infinitely extended planar membrane that is stretched between e.g. two parallel fivebranes corresponds to a BPS state that preserves one quarter of the original supersymmetries to leading order [14]. It was argued in [14,15] that the boundary dynamics of the open membrane is described by a six-dimensional non-perturbative superstring theory [30]. This string becomes tensionless as the two fivebranes approach each other. The boundary conditions for the bosonic fields of this open membrane are described by two equations of the type (2.15), one for each boundary. The fermionic fields obey the boundary conditions (2.19) and (2.22). Thus half of the original field Θ propagates on the fivebrane and that half has a definite $SO(4)$ chirality in light-cone gauge. Upon double dimensional reduction to ten dimensions along the fivebranes, we are left with a string stretched between two Dirichlet four-branes of the Type IIA string theory.

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Appendix A: Boundary Conditions in Siegel Gauge

For completeness we want to show how to get the full $SO(5, 1)$ chirality for an open extended membrane. For this purpose it will be convenient to work in the so-called Siegel gauge for the fermionic field [18]. The κ -symmetry can be used to set

$$P_+ \Theta = 0. \quad (4.1)$$

We are then left with 16 physical components for Θ . Now we want to determine the boundary conditions for Θ . The vanishing of $\delta\mathcal{S}$ under the variation of the fermionic field requires

$$\int_{\partial\Sigma} \bar{\Theta} \not{n} \delta\Theta = 0, \quad (4.2)$$

where $\not{n} = n^\alpha \partial_\alpha X^M_{\mathcal{D}} \Gamma_M$. The expression (4.2) vanishes if we impose the Dirichlet condition

$$(1 + \prod_{M \in \mathcal{N}} \Gamma_M) \Theta = 0 \quad \text{on } \partial\Sigma. \quad (4.3)$$

This equation implies that the fermionic fields have a well defined $SO(5, 1)$ chirality, longitudinal to the fivebrane.

The boundary conditions for the orthogonal component of the fermionic field can be obtained from the vanishing of the boundary term

$$\int_{\partial\Sigma} \delta X^M_{\mathcal{N}} \bar{\Theta} \Gamma_M n^\alpha \partial_\alpha \Theta = 0. \quad (4.4)$$

which leads us to

$$n^\alpha \partial_\alpha (\not{\varphi} \Theta) = 0 \quad \text{on } \partial\Sigma. \quad (4.5)$$

This boundary condition can be obtained out of the Dirac equation and the boundary condition (4.3).

Appendix B: Notation and Useful Formulas

▷ The space-time gamma matrices satisfy the Clifford algebra

$$\{\Gamma^M, \Gamma^N\} = 2\eta^{MN},$$

with signature $(-, +, \dots, +)$. Γ^0 is antihermitian and the rest of the gamma matrices are hermitian with

$$\Gamma^{M\dagger} = \Gamma^0 \Gamma^M \Gamma^0$$

and for an arbitrary spinor we define $\bar{\Psi} = \Psi^\dagger \Gamma^0$. We use the notation

$$\Gamma^{M_1 \dots M_n} = \Gamma^{[M_1} \Gamma^{M_2} \dots \Gamma^{M_n]},$$

where the square bracket implies a sum over $n!$ terms with an $1/n!$ prefactor.

▷ In the light-cone gauge we use the notation

$$X^\pm = \frac{1}{\sqrt{2}} (X^0 \pm X^{10}) \quad \text{and} \quad \Gamma^\pm = \frac{1}{\sqrt{2}} (\Gamma^0 \pm \Gamma^{10}),$$

with $(\Gamma^\pm)^2 = 0$, $\{\Gamma^+, \Gamma^-\} = -2$. The gamma matrices are decomposed according to

$$\Gamma^+ = I_{16} \otimes \begin{pmatrix} 0 & 0 \\ \sqrt{2}i & 0 \end{pmatrix}, \quad \Gamma^- = I_{16} \otimes \begin{pmatrix} 0 & \sqrt{2}i \\ 0 & 0 \end{pmatrix}, \quad \Gamma^I = \gamma^I \otimes \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

and they satisfy $\{\gamma^I, \gamma^J\} = 2\delta^{IJ}$. Set $\Theta = (i\Theta_1, \Theta_2)$ and $\bar{\Theta} = (-i\bar{\Theta}_2, -\bar{\Theta}_1)$. We use the “flip” rule:

$$\bar{\theta} \gamma^{\nu_1 \dots \nu_n} \psi = (-)^{\frac{n(n-1)}{2} + 1} \bar{\psi} \gamma^{\nu_1 \dots \nu_n} \theta.$$

▷ A useful identity is

$$\Gamma \Gamma_M \partial^\gamma X^M = \partial^\gamma X^M \Gamma_M \Gamma = \frac{1}{2} \epsilon^{\alpha\beta\gamma} \partial_\alpha X^R \partial_\beta X^S \Gamma_{RS}.$$

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